Vectors

Assertion & Reason Type Questions

Directions: In the following questions, each question contains Assertion (A) and Reason (R). Each question has 4 choices (a), (b), (c) and (d) out of which only one is correct. The choices are:

a. Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of Assertion (A)

b. Both Assertion (A) and Reason (R) are true but Reason (R) is not the correct explanation of Assertion (A)

c. Assertion (A) is true but Reason (R) is false

d. Assertion (A) is false but Reason (R) is true

Q1.

Assertion (A): The magnitude of the resultant of vectors $\vec{a} = 2\hat{i} + \hat{j} + \hat{k}$ and $\vec{b} = \hat{i} + 2\hat{j} + 3\hat{k}$ is $\sqrt{34}$.

Reason (R): The magnitude of a vector can never be negative.

Answer : (b) Both Assertion (A) and Reason (R) are true but Reason (R) is not the correct explanation of Assertion (A)

Q2.

Assertion (A): The unit vector in the direction of sum of the vectors $\hat{i} + \hat{j} + \hat{k}$, $2\hat{i} - \hat{j} - \hat{k}$ and $2\hat{j} + 6\hat{k}$ is $-\frac{1}{7}(3\hat{i} + 2\hat{j} + 6\hat{k})$.

Reason (R): Let $\vec{a}$ be a non-zero vector, then $\frac{\vec{a}}{|\vec{a}|}$ is a unit vector parallel to $\vec{a}$.

Answer : (d) Assertion (A) is false but Reason (R) is true

Q3.

Assertion (A): If the points $\vec{P}=(\vec{a}+\vec{b}-\vec{c})$, $\vec{Q}=(2\vec{a}+\vec{b})$ and $\vec{R}=(\vec{b}+t\vec{c})$ are collinear, where $\vec{a},\vec{b},\vec{c}$ are three non-coplanar vectors, then the value of t is -2.

Reason (R): If P, Q, R are collinear, then $\overrightarrow{PQ} \parallel \overrightarrow{PR}$ or $\overrightarrow{PQ}=\lambda\overrightarrow{PR}$, $\lambda \in R$.

Answer : (a) Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of Assertion (A)

Q4.

Assertion (A): The adjacent sides of a parallelogram are along $\vec{a}=\hat{i}+2\hat{j}$ and $\vec{b}=2\hat{i}+\hat{j}$. The angle between the diagonals is 150°.

Reason (R): Two vectors are perpendicular to each other if their dot product is zero.

Answer : (d) Assertion (A) is false but Reason (R) is true

Q5.

Assertion (A): If $\vec{a}+\vec{b}+\vec{c}=\vec{0}$, $|\vec{a}|=3$, $|\vec{b}|=4$, $|\vec{c}|=5$, then $\vec{a}\cdot\vec{b}+\vec{b}\cdot\vec{c}+\vec{c}\cdot\vec{a}$ is equal to -25.

Reason (R): If $\vec{a}+\vec{b}+\vec{c}=\vec{0}$, then the angle θ between $\vec{b}$ and $\vec{c}$ is given by $\cos\theta=\dfrac{\vec{a}^2-\vec{b}^2-\vec{c}^2}{2\vec{b}\vec{c}}$.

Answer : (b) Both Assertion (A) and Reason (R) are true but Reason (R) is not the correct explanation of Assertion (A)

Q6.

Assertion (A): The length of projection of the vector $3\hat{i}-\hat{j}-2\hat{k}$ on the vector $\hat{i}+2\hat{j}-3\hat{k}$ is $\frac{7}{\sqrt{14}}$.

Reason (R): The projection of a vector $\vec{a}$ on another vector $\vec{b}$ is $\frac{(\vec{a}\cdot\vec{b})}{|\vec{b}|}$.

Answer : (a) Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of Assertion (A)

Q7.

Let $\vec{a}$ and $\vec{b}$ be proper vectors and θ be the angle between them.

Assertion (A): $(\vec{a}\times\vec{b})^2+(\vec{a}\cdot\vec{b})^2 \neq (\vec{a})^2\ (\vec{b})^2$

Reason (R): $\sin^2\theta+\cos^2\theta=1$

Answer : (d) Assertion (A) is false but Reason (R) is true

Q8.

Assertion (A): If $\vec{a}=\hat{i}-\hat{j}+\hat{k}$, $\vec{a}\cdot\vec{b}=0$ and $\vec{a}\times\vec{b}=\vec{c}$, where $\vec{c}=-2\hat{i}-\hat{j}+\hat{k}$, then $\vec{b}=(0,1,1)$.

Reason (R): If $\vec{a}=x_1\hat{i}+y_1\hat{j}+z_1\hat{k}$ and $\vec{b}=x_2\hat{i}+y_2\hat{j}+z_2\hat{k}$, then

$$\vec{a}\times\vec{b}=\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ x_2 & y_2 & z_2\\ x_1 & x_2 & z_1\end{vmatrix}.$$

Answer : (c) Assertion (A) is true but Reason (R) is false

Q9.

Assertion (A): If $(\vec{a} \times \vec{b})^2 + (\vec{a} \cdot \vec{b})^2 = 400$ and $|\vec{a}| = 4$, then $|\vec{b}| = 9$.

Reason (R): If $\vec{a}$ and $\vec{b}$ are any two vectors, then $(\vec{a} \times \vec{b})^2 = (\vec{a})^2 \ (\vec{b})^2 - (\vec{a} \cdot \vec{b})^2$.

Answer : (d) Assertion (A) is false but Reason (R) is true

Assertion (A): The position of a particle in a rectangular coordinate system is (3, 2, 5). Then its position vector be $2\hat{i}+5\hat{j}+3\hat{k}$.

Reason (R): The displacement vector of the particle that moves from point $P(2, 3, 5)$ to point $Q(3, 4, 5)$ is $\hat{i}+\hat{j}$.

Ans. Option (D) is correct.

Explanation: Assertion (A) is wrong.

The position of a particle in a rectangular coordinate system is (3, 2, 5). Then its position vector be $3\hat{i}+2\hat{j}+5\hat{k}$.

Reason (R) is correct.

The displacement vector of the particle that moves from point $P(2, 3, 5)$ to point $Q(3, 4, 5)$

$$= (3-2)\hat{i}+(4-3)\hat{j}+(5-5)\hat{k}$$
$$= \hat{i}+\hat{j}$$

Assertion (A): The direction of cosines of vector $\vec{A}=2\hat{i}+4\hat{j}-5\hat{k}$ are $\frac{2}{\sqrt{45}}, \frac{4}{\sqrt{45}}, -\frac{5}{\sqrt{45}}$.

Reason (R): A vector having zero magnitude and arbitrary direction is called 'zero vector' or 'null vector'.

Ans. Option (B) is correct.

Explanation: Assertion (A) is correct.

Direction cosines of $\vec{A}=2\hat{i}+4\hat{j}-5\hat{k}$ are :

$$\frac{2}{\sqrt{2^2+4^2+(-5)^2}}, \frac{4}{\sqrt{2^2+4^2+(-5)^2}}, \frac{-5}{\sqrt{2^2+4^2+(-5)^2}}$$

Or, $\frac{2}{\sqrt{45}}, \frac{4}{\sqrt{45}}, \frac{-5}{\sqrt{45}}$